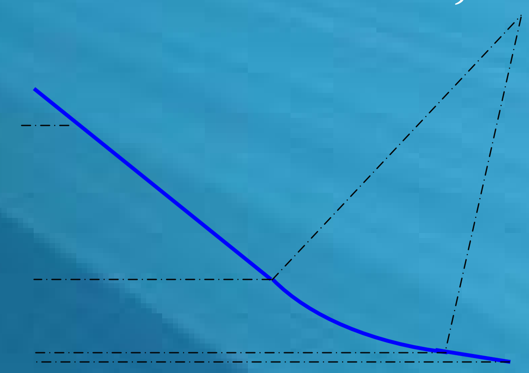
Prediction of performances in ski-jumping

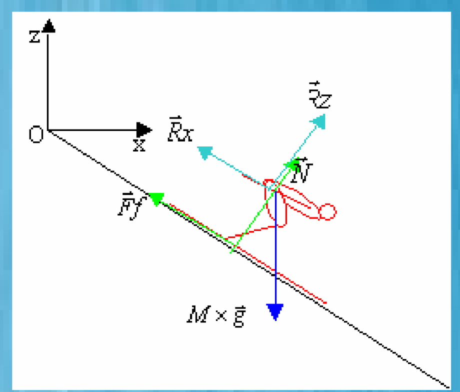
-in-run and take-off phases modeling-

Karine Lamy[1 & 2], Michel Perraudin[2], Nicolas Coulmy[3], Frédérique Hintzy[1]

Contact : info@cmefe.ch

Optimization of technique to improve performance in ski jumping is a complex process to which wind tunnel can contribute data and understanding. In ski jumping, take-off action is one of most important factor. Coaches and athletes are confronted with the problem of assessing the efficiency of different motion strategies. Mathematics models may provide an adequate solution to confronte objectives data and athletes perception in condition of wind. The purpose of this study is conducted to develop a simplified mathematical model of in-run and take-off performance in ski jumping as a support for training session of athlètes in wind tunnel. this model based on computing each of the forces involved in the motion's equation. This model integrate the influence of the lift force onto the friction force and the jumper's inertia in the changes of hill's curve. Wind tunnel equipment allows to measure the external forces exerted on the ski-skier system , the motion's equation can be solved and simulations can be performed. These can be used to estimate variations in performance induced by different postural strategies. Such simulations find an application in the field of training as they permit to assess the impact on performance of a given strategy compared with another.





Equations part 1:

$$M\frac{dV_x}{dt} = -(N+Rz)\times\sin\varphi - (Rx+Ff)\times\cos\varphi \quad ①$$

$$M\frac{dV_z}{dt} = (N+Rz)\times\cos\varphi - (Ff+Rx)\times\sin\varphi - Mg \quad ②$$

$$N = M\times g\times\cos(\varphi) - Rz \qquad \text{car } r=\infty \quad ③$$

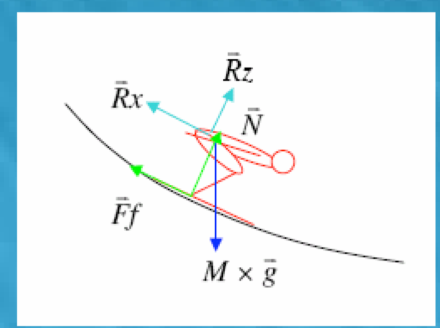
$$Vx = \frac{dx}{dt} \quad ④$$

$$Vz = \frac{dz}{dt} \quad ⑤$$

$$\dot{\varphi} = \dot{\gamma} \quad ⑥$$

Initial conditions:

X0	=	0		
Z0	=	0		
V0	=	0		
with Vx initial	=	$V0\times\cos(\varphi)$	$=0$	
Vz initial	=	$V0\times\sin(\varphi)$	$=0$	



Equations part 2:

$$M\frac{dV_x}{dt} = -(N+Rz)\times\sin\varphi - (Rx+Ff)\times\cos\varphi \quad ①$$

$$M\frac{dV_z}{dt} = (N+Rz)\times\cos\varphi - (Ff+Rx)\times\sin\varphi - Mg \quad ②$$

$$N = M\times g\times\cos(\varphi) - Rz + \left|\frac{M\times V^2}{r}\right| \quad ③$$

$$Vx = \frac{dx}{dt} \quad ④$$

$$Vz = \frac{dz}{dt} \quad ⑤$$

$$\varphi = \arctan\left(\frac{Vz}{Vx}\right) \quad ⑥$$

Initial conditions:

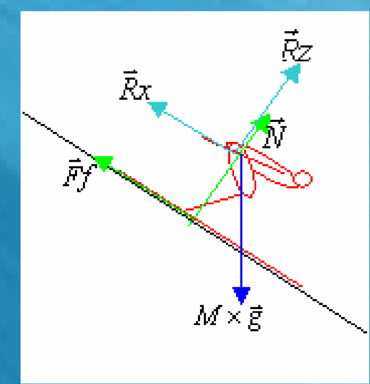
Kinetic energy end of part 1 = kinetic energy beginning part 2

$$\frac{M\times V_{1\,finale}^2}{2} + \frac{I\times\omega_{1\,finale}^2}{2} = \frac{M\times V_{2initiale}^2}{2} + \frac{I\times\omega_{2initiale}^2}{2}$$

$$\frac{I\times\omega_{1\,finale}^2}{2} = 0$$

$$\frac{M\times V_{1\,finale}^2}{2} = \frac{M\times V_{2initiale}^2}{2} + \frac{I\times\omega_{2initiale}^2}{2} \qquad \text{with} \qquad \omega_{2initiale} = \frac{V_{2initiale}}{r}$$

$$\left|V_{2initiale}\right| = \sqrt{\frac{V_{1\,finale}^2}{1+\frac{I}{M\times r^2}}}$$



Equations part 3:

$$M\frac{dV_x}{dt} = -(N+Rz)\times\sin\varphi - (Rx+Ff)\times\cos\varphi \quad ①$$

$$M\frac{dV_z}{dt} = (N+Rz)\times\cos\varphi - (Ff+Rx)\times\sin\varphi - Mg \quad ②$$

$$N = M\times g\times\cos(\varphi) - Rz \qquad \text{car } r=\infty \quad ③$$

$$Vx = \frac{dx}{dt} \quad ④$$

$$Vz = \frac{dz}{dt} \quad ⑤$$

$$\varphi = \hat{\alpha} \quad ⑥$$

Initial conditions:

Kinetic energy end of part 2 = kinetic energy beginning part 3

$$\frac{M\times V_{2\,finale}^2}{2} + \frac{I\times\omega_{2\,finale}^2}{2} = \frac{M\times V_{3initiale}^2}{2} + \frac{I\times\omega_{3initiale}^2}{2}$$

$$\frac{I\times\omega_{3initiale}^2}{2} = 0$$

$$\frac{M\times V_{3initiale}^2}{2} = \frac{M\times V_{2\,finale}^2}{2} + \frac{I\times\omega_{2\,finale}^2}{2} \qquad \text{with} \qquad \omega_{2\,finale} = \frac{V_{2\,finale}}{r}$$

$$\left|V_{3initiale}\right| = \sqrt{V_2^2\times\left(1+\frac{I}{M\times r^2}\right)}$$

Equations part 4:

$$M\frac{dV}{dt}(x) = -Rz\times\sin\varphi - Rx\times\cos\varphi \quad ①$$

$$M\frac{dV}{dt}(z) = Rz\times\cos\varphi - Rx\times\sin\varphi - Mg \quad ②$$

$$Vx = \frac{dx}{dt} \quad ④$$

$$Vz = \frac{dz}{dt} \quad ⑤$$

$$\varphi = \arctan\left(\frac{Vz}{Vx}\right) \quad ⑥$$

Initial conditions:

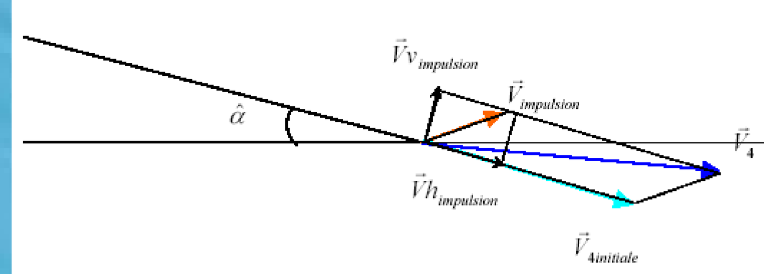
Kinetic energy end of part 3 = kinetic energy beginning part 4

$$\frac{M\times V_{3\,finale}^2}{2} + \frac{I\times\omega_{3\,finale}^2}{2} = \frac{M\times V_{4initiale}^2}{2} + \frac{I\times\omega_{4initiale}^2}{2}$$

$$\frac{I\times\omega_{3\,finale}^2}{2} = 0$$

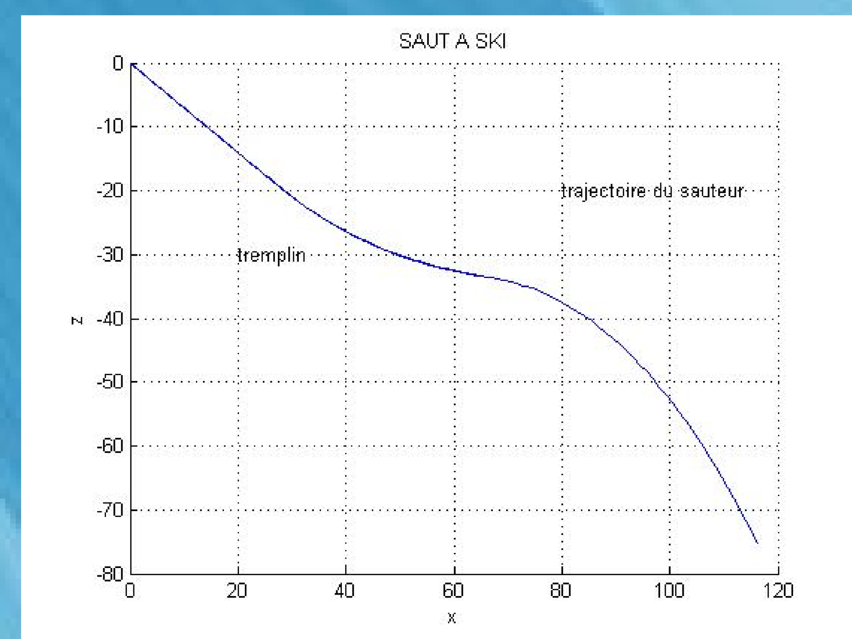
$$\frac{M\times V_{3\,finale}^2}{2} = \frac{M\times V_{4initiale}^2}{2} + \frac{I\times\omega_{4initiale}^2}{2}$$

$$\left|V_{4\,initiale}\right| = \sqrt{V_{3\,finale}^2 - \frac{I\times\omega_{4\,initiale}^2}{M}}$$



$$\begin{cases} V_{4\,sur\,x} = \left(Vh_{impulsion} + V_{4\,initiale}\right)\times\cos\hat{\alpha} \\ V_{4\,surz} = Vv_{impulsion}\times\cos\hat{\alpha} - V_{4\,initiale}\times\sin\hat{\alpha} \end{cases}$$

The resolution of the equations was carried with Matlab:



(2) HES-SO// Genève
(3)
(1)